

On Existences Of Periodic Orbits For Hamilton Systems*

Renyi Ma

Department of Mathematics

Tsinghua University

Beijing, 100084

People's Republic of China

rma@math.tsinghua.edu.cn

Abstract

In this article, we prove that either there exists at least one periodic orbit of Hamilton vector field on a given energy hypersurface in R^{2n} or there exist at least two periodic orbits on the near-by energy hypersurface in R^{2n} . The more general results are also obtained.

Keywords Symplectic geometry, J-holomorphic curves, Periodic Orbit.

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1 Introduction and results

Let Σ be a smooth closed oriented manifold of dimension $2n - 1$ in R^{2n} , here $(R^{2n}, \omega_0)(\omega_0 = \sum_{i=1}^n dx_i \wedge dy_i)$ is the standard symplectic space. Then there exists a unique vectorfield X_Σ so called Hamilton vector field defined by $i_{X_\Sigma} \omega_0|_\Sigma \equiv 0$. A periodic Hamilton orbit in Σ is a smooth path $x : [0, T] \rightarrow \Sigma$, $T > 0$ with $\dot{x}(t) = X_\Sigma(x(t))$ for $t \in (0, T)$ and $x(0) = x(T)$. Seifert in [22] raised the following conjecture:

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Conjecture(see[22]). Let ω_0 be the standard symplectic form on R^{2n} . Let Σ be a closed $(2n - 1)$ -hypersurface in R^{2n} . Then there is a closed Hamilton periodic orbit in Σ .

Rabinowitz [21] and Weinstein [25, 26] proved that if S is starshape resp. convex, SC holds. Weinstein conjectured SC holds for the hypersurface of contact type in general symplectic manifold(WC). In R^{2n} , SC implies WC . Viterbo [24] proved the WC in R^{2n} . By the Viterbo's work, Hofer and Zehnder in [13] proved the near-by SC holds. Struwe in [23] proved that SC almost holds by modifying Hofer-Zehnder's work. Ginzburg in [7] and Herman in [10] gave a counter-example [10] for the SC . After Viterbo's work, many results were obtained by using variational method or Gromov's J -holomorphic curves via nonlinear Fredholm alternative, see [5, 11, 12, 13, 15, 16, 17, 18] etc.

Let (M, ω) be a symplectic manifold and $h(t, x)(= h_t(x))$ a compactly supported smooth function on $M \times [0, 1]$. Assume that the segment $[0, 1]$ is endowed with time coordinate t . For every function h define the (*time-dependent*) *Hamiltonian vector field* X_{h_t} by the equation:

$$dh_t(\eta) = \omega(\eta, X_{h_t}) \quad \text{for every } \eta \in TM \quad (1.1)$$

The flow g_h^t generated by the field X_{h_t} is called *Hamiltonian flow* and its time one map g_h^1 is called *Hamiltonian diffeomorphism*.

Now assume that H be a time independent smooth function on M and X_H its induced vector field.

Let (M, ω) be a symplectic manifold. Let J be the almost complex structure tamed by ω , i.e., $\omega(v, Jv) > 0$ for $v \in TM$. Let \mathcal{J} the space of all tame almost complex structures.

Definition 1.1 *Let*

$$s(M, \omega, J) = \inf \left\{ \int_{S^2} f^* \omega > 0 \mid f : S^2 \rightarrow M \text{ is } J\text{-holomorphic} \right\}$$

Definition 1.2 *Let*

$$s(M, \omega) = \sup_{J \in \mathcal{J}} l(M, \omega, J)$$

Let W be a Lagrangian submanifold in M , i.e., $\omega|W = 0$.

Definition 1.3 Let

$$l(M, W, \omega) = \inf \left\{ \left| \int_{D^2} f^* \omega \right| > 0 \mid f : (D^2, \partial D^2) \rightarrow (M, W) \right\}$$

Theorem 1.1 Let (M, ω) be a closed compact symplectic manifold or a manifold convex at infinity and $M \times C$ be a symplectic manifold with symplectic form $\omega \oplus \sigma$, here (C, σ) standard symplectic plane. Let $2\pi r_0^2 < s(M, \omega)$ and $B_{r_0}(0) \subset C$ the closed ball with radius r_0 . Assume that H be a time independent smooth function on $M \times C$ and X_H its induced vector field. If $\Sigma = H^{-1}(c)$ be a smooth hypersurface in $M \times B_{r_0}(0)$, X_H its Hamilton vector field, then either there exists at least one periodic orbits of X_H on Σ or there exists at least two periodic orbits of X_H on $\Sigma' = H^{-1}(c')$, c' is close to c as one wants.

Theorem 1.2 Let (M, ω) be an exact symplectic manifold, i.e., $\omega = d\alpha$ for some 1-form α . Assume that H be a time independent smooth function on M and X_H its induced vector field. If $\Sigma = H^{-1}(c)$ be a smooth compact hypersurface in M and there exists a Hamiltonian diffeomorphism h such that $h(\Sigma) \cap \Sigma = \emptyset$, then either there exists at least one periodic orbits of X_H on Σ or there exists at least two periodic orbits of X_H on $\Sigma' = H^{-1}(c')$, c' is close to c as one wants.

The near-by and almost existence results or the finity of Hofer-Zehnder's for bounded set in the above symplectic manifolds can be obtained as in [14]. For example, one has

Corollary 1.1 Let M be any open manifold and $(T^*M, d\alpha)$ be its cotangent bundle. Assume that B is bounded set in T^*M , then the Hofer-Zehnder capacity $C_{HZ}(B)$ is finite.

Theorem1.1-1.2 was reported in the proceedings of the international conference on “Boundary Value Problems, Integral Equations, And Related Problems”(5-13 August 2002); “ICM2002-Beijing Satellite Conference on Nonlinear Functional Analysis, August 14-18,2002 Taiyuan. Theorem 1.1-1.2 can be generalized to the products of symplectic manifolds in Theorem 1.1-1.2. The proofs of Theorem1.1-1.2 is close as in [17, 19]. Here we flow the Monke’s method in [19]. If a $(n - 1)$ -dimensional submanifold \mathcal{L} in Σ satisfying that \mathcal{L} is transversal to the hamilton vector field X_H and $\omega_0|_{\mathcal{L}} = 0$ and

$\omega_0|_{\pi_2(M, \mathcal{L})} = 0$, then we call \mathcal{L} the Hamilton-Legendre isotropic submanifold. A Hamilton-Arnold chord in Σ is a smooth path $x : [0, T] \rightarrow \Sigma, T > 0$ with $\dot{x}(t) = X_\Sigma(x(t))$ for $t \in (0, T)$ and $x(0), x(T) \in \mathcal{L}$. Then one can also prove the Hamilton's chord almost existence results as Theorem 1.1-1.2.

2 Lagrangian Non-Squeezing

Theorem 2.1 ([20]) Let (M, ω) be a closed compact symplectic manifold or a manifold convex at infinity and $M \times C$ be a symplectic manifold with symplectic form $\omega \oplus \sigma$, here (C, σ) standard symplectic plane. Let $2\pi r_0^2 < s(M, \omega)$ and $B_{r_0}(0) \subset C$ the closed disk with radius r_0 . If W is a close Lagrangian manifold in $M \times B_{r_0}(0)$, then

$$l(M, W, \omega) < 2\pi r_0^2$$

This can be considered as an Lagrangian version of Gromov's symplectic non-squeezing ([8]).

Corollary 2.1 (Gromov [8]) Let (V', ω') be an exact symplectic manifold with restricted contact boundary and $\omega' = d\alpha'$. Let $V' \times C$ be a symplectic manifold with symplectic form $\omega' \oplus \sigma = d\alpha = d(\alpha' \oplus \alpha_0)$, here (C, σ) standard symplectic plane. If W is a close exact Lagrangian submanifold, then $l(V' \times C, W, \omega) = \infty$, i.e., there does not exist any close exact Lagrangian submanifold in $V' \times C$.

Corollary 2.2 Let L^n be a close Lagrangian in R^{2n} and $l(R^{2n}, L^n, \omega) = 2\pi r_0^2 > 0$, then L^n can not be embedded in $B_{r_0}(0)$ as a Lagrangian submanifold.

3 Construction Of Lagrangian

3.1 First Case: no periodic orbit

Let $(V, \omega) = (R^{2n} \times R^{2n}, \omega_0 \oplus \omega_0)$ be the standard symplectic vector space, here $\omega_0 = d\lambda_0 = d(\frac{1}{2}(x_i dy_i - y_i dx_i))$. Let Σ be a oriented closed hypersurface in R^{2n} . Let $\mathcal{L} = \{(\sigma, \sigma) | \sigma \in \Sigma \subset R^{2n}\}$ be a closed isotropic submanifold contained in $(\Sigma', \omega) = (\Sigma \times \Sigma, \omega_0 \oplus \omega_0)$, i.e., $Q^* \omega|_{\mathcal{L}} = 0$. Since Σ is oriented

in R^{2n} , the normal bundle of Σ is trivial. So, the tubular neighbourhood $Q_\delta(\Sigma)$ of Σ is foliated by Σ . We now define a Hamiltonian as follows. Define $Q_\delta(\Sigma) = \cup_{|t| \leq \delta} \psi_t(\Sigma)$, here ψ_t is the flow of the normal vector field of Σ . Define $h(x) = t(x)$, $t(x)$ is the arrival time of ψ_t from x to Σ for $x \in Q_\delta(\Sigma)$. Let $H(\sigma_1, \sigma_2) = h(\sigma_2)$ on $\bar{Q}_\delta = Q_\delta(\Sigma) \times Q_\delta(\Sigma)$ and X_H be its Hamilton vectorfield. Let η_s be the Hamilton flow on \bar{Q}_δ induced by X_H . Let S be a smooth compact hypersurface in R^{4n} which intersects the hypersurface $E = R^{2n} \times \Sigma$ transversally and contains \mathcal{L} . Furthermore, S is transversal to the hamilton vector field X_H . Let $s(x)$ be the time to S of hamilton flow η_s which is well defined in the neighbourhood $U_{\delta_1}(S)$. Let X_S be the Hamilton vector field of S on U_{δ_1} and ξ_t be the flow of X_S defined on U_{δ_1} . Since $X_S(H) = \{s, H\} = -\{H, s\} = -X_H(s) = -1 \neq 0$, here $\{\cdot, \cdot\}$ is the Poisson bracket, so X_S is transversal to E . Then, there exists δ_0 such that $\xi_t(x)$ exists for any $x \in U_{\delta_0}(S \cap E)$, $|t| \leq 100\delta_0$. Let $L_0 = \cup_{t \leq \delta_0} \xi_t(\mathcal{L})$. One has

Lemma 3.1 L_0 is a Lagrangian submanifold in (R^{4n}, ω) .

Proof. Let

$$\begin{aligned} F : [-\delta_0, \delta_0] \times \mathcal{L} &\rightarrow \bar{Q}_\delta \\ F(t, l) &= \xi_t(l). \end{aligned} \tag{3.1}$$

Then,

$$\begin{aligned} F^* \omega &= F_t^* \omega + i_{X_S} \Omega \wedge dt \\ &= 0 - (ds|S) \wedge dt = 0 \end{aligned} \tag{3.2}$$

This checks that $L_0 = F([-\delta_0, \delta_0]) \times \mathcal{L}$ is Lagrangian submanifold.

Lemma 3.2 Let $M = E \cap S$ and $\omega_M = \omega|_M$. Then there exist $\delta_0 > 0$ and a neighbourhood M_0 of \mathcal{L} in M such that $G : M_0 \times [-\delta_0, \delta_0] \times [-\delta_0, \delta_0] \rightarrow R^{4n}$ defined by $G(m, t, s) = \eta_s(\xi_t(m))$ is an symplectic embedding.

$$G^* \omega = \omega_M + dt \wedge ds \tag{3.3}$$

Proof. It is obvious.

Let $U_T = G(M_T \times [-\delta_0, \delta_0] \times [-T, T])$. If there does not exist periodic solution in $Q_\delta(\Sigma)$ and M_T is a very small neighbourhood of \mathcal{L} , then $s(x)$

and X_S is well defined on U_T . Therefore, there exists δ_T such that the flow $\xi_t(x)$ of X_S exists for any $x \in U_T$, $|t| \leq 100\delta_T$. Let $U_0 = U_{\delta_0}(S \cap E) \cap U_T$, $U_k = \eta_{k\delta_0}(U_0) \subset U_T$, $k = 1, \dots, k_T$. Let $X_k = X_S|U_k$. Let $\bar{X}_k = \eta_{k\delta_0*}X_0$. We claim that $\bar{X}_k = X_k$. Since $s(x)$ and $H(x)$ is defined on U_T and $\{H, s\} = 1$, so $\xi_t(\eta_s(x)) = \eta_s(\xi_t(x))$ for $x \in U_T$. Differentiate it, we get $X_S(\eta_s(x)) = \eta_{s*}X_S(x)$. Take $s = k$, one proves $\bar{X}_k = X_k$. Recall that the flow $\xi_t(x)$ of X_S exists for any $x \in U_{\delta_0}(S \cap E)$, $|t| \leq 100\delta_0$. So, the flow $\bar{\xi}_t^k(x)$ of \bar{X}_k exists for any $x \in U_k$, $|t| \leq 100\delta_0$. Therefore, the flow $\xi_t^k(x)$ of X_k exists for any $x \in U_k$, $|t| \leq 100\delta_0$. This Proves that the flow $\xi_t(x)$ of X_S exists for any $x \in U_T$, $|t| \leq 100\delta_0$.

Theorem 3.1 (*Long Darboux theorem*) *Let $M = E \cap S$, $\omega_M = \omega|M$. Let M_0 as in Lemma 3.2. Let $(U'_T, \omega') = (M_T \times [-\delta_0, \delta_0] \times [-T, T], \omega_M + dH' \wedge ds')$. If there does not exist periodic solution in $Q_\delta(\Sigma)$, then there exists a symplectic embedding $G : U'_T \rightarrow \bar{Q}_\delta$ defined by $G(m, H', s') = \eta_{s'}(\xi_{H'}(m))$ such that*

$$G^*\omega = \omega_M + dH' \wedge ds'. \quad (3.4)$$

Proof. We follow the Arnold's proof on Darboux's theorem in [1]. Take a Darboux chart U on M , we assume that $\omega_M| = \sum_{i=1}^{2n-1} dp'_i \wedge dq'_i$. Now computing the Poisson brackets $\{\cdot, \cdot\}^*$ of $(p'_1, q'_1; \dots, p'_{2n-1}, q'_{2n-1}; H', s')$ for $G^*\omega$ on U'_T . Let $P'_i(G(p'_1, q'_1; \dots, p'_{2n-1}, q'_{2n-1}; H', s')) = P'_i(\xi_{H'}\eta_{s'}(p', q')) = p'_i$, and $Q'_i(G(p'_1, q'_1; \dots, p'_{2n-1}, q'_{2n-1}; H', s')) = Q'_i(\xi_{H'}\eta_{s'}(p', q')) = q'_i$, $i = 1, \dots, 2n - 1$. $\{H', s'\}^* = G^*\omega(\frac{\partial}{\partial H'}, \frac{\partial}{\partial s'}) = \omega(G_*\frac{\partial}{\partial H'}, G_*\frac{\partial}{\partial s'}) = \omega(X_H, X_S) = 1$, $\{H', p'_i\}^* = G^*\omega(\frac{\partial}{\partial H'}, \frac{\partial}{\partial p'_i}) = \omega(G_*\frac{\partial}{\partial H'}, G_*\frac{\partial}{\partial p'_i}) = \omega(X_H, X_{P'_i}) = -X_{P'_i}(H) = 0$. Similarly, $\{H', q'_i\}^* = 0$. Similarly, $\{s', H'\}^* = 1$, $\{s', p'_i\}^* = 0$, $\{s', q'_i\}^* = 0$. Note that $\omega = \xi_t^*\eta_s^*\omega$, so $\{p'_i, q'_j\}^* = G^*\omega(\frac{\partial}{\partial p'_i}, \frac{\partial}{\partial q'_j}) = \omega(G_*\frac{\partial}{\partial p'_i}, G_*\frac{\partial}{\partial q'_j}) = \omega(\xi_{H'}\eta_{s'}\frac{\partial}{\partial p'_i}, \xi_{H'}\eta_{s'}\frac{\partial}{\partial q'_j}) = \xi_{H'}^*\eta_{s'}^*\omega(\frac{\partial}{\partial p'_i}, \frac{\partial}{\partial q'_j}) = \omega(\frac{\partial}{\partial p'_i}, \frac{\partial}{\partial q'_j}) = \omega_M(\frac{\partial}{\partial p'_i}, \frac{\partial}{\partial q'_j}) = \delta_{ij}$. This shows that the Poisson brackets $\{\cdot, \cdot\}^*$ is same as the Poisson brackets $\{\cdot, \cdot\}'$ for ω' . So, $\omega' = G^*\omega$. This finishes the proof.

Take a disk M_0 enclosed by the circle E_0 which is parametrized by $t \in [0, \delta_0]$ in $(s', H') - plane$ such that $M_0 \subset [-2s_0, 2s_0] \times [0, \varepsilon]$ and $area(M_0) \geq 2s_0\varepsilon$. Now one checks that $L = G(\mathcal{L} \times E_0)$ satisfy

$$\omega|L = G^*\omega(\mathcal{L} \times E_0) = dH' \wedge ds'|E_0 = 0. \quad (3.5)$$

So, L is a Lagrangian submanifold.

Lemma 3.3 *If there does not exist any periodic orbit in (Q_δ, X_H) , then L is a close Lagrangian submanifold. Moreover*

$$l(V, L, \omega) = \text{area}(M_0) \quad (3.6)$$

Proof. It is obvious that F is a Lagrangian embedding. If the circle C homotopic to $C_1 \subset \mathcal{L} \times s_0$ then we compute

$$\int_C F^*(p_i dq_i) = \int_{C_1} F^*(p_i dq_i) = 0. \quad (3.7)$$

since $\lambda|C_1 = 0$ due to $C_1 \subset \mathcal{L}$ and \mathcal{L} is “Legendre submanifold”.

If the circle C homotopic to $C_1 \subset l_0 \times S^1$ then we compute

$$\int_C F^*(p_i dq_i) = \int_{C_1} F^*(p_i dq_i) = n(\text{area}(M_0)). \quad (3.8)$$

This proves the Lemma.

3.2 Second case: single curve of periodic orbits

Theorem 3.2 (*Long Darboux cover theorem*) *Let M_0 be as in Lemma 3.2. Let $\bar{U}'_T = M_0 \times [-\delta_0, \delta_0] \times [-T, T]$, $\omega' = \omega_M + dH' \wedge ds'$. Then there exists a symplectic immersion $G : U'_T \rightarrow \bar{Q}_\delta$ satisfies $G(m; H', s') = \xi_{H'} \eta_{s'}(m)$; and*

$$G^*\omega = \omega_M + dH' \wedge ds'. \quad (3.9)$$

Proof. By the proof of Theorem 3.1.

Now Assume that H is “single”, i.e., there exist only one family of periodic orbits, more precisely, the periodic orbits consist of $x(t, (\sigma_0, c))$ for some $0 < c < \delta$ in Q_δ such that $H(x(t, (\sigma, c))) = c$ with period T_c . Now we assume that there does not exist any peiodic orbit on $H^{-1}(0)$. Let $Z'_T = \mathcal{L} \times [-\delta_0, \delta_0] \times [-T, T]$. $Z_T = G(Z'_T)$. Let $\gamma(l) = L_0 \cap (Z_T \setminus L_0)$ and $\{\gamma'_i\}_{i=1}^m = G^{-1}(\gamma) \subset Z'_T$. We claim that one still can take a disk M'_0 enclosed by the circle E'_0 which is parametrized by $t \in [0, t_0]$ in $(s', H') - \text{plane}$ such that $G|E'_0$ is an embedding and $M'_0 \subset [-2s_0, 2s_0] \times [0, \varepsilon]$ with $0 < \varepsilon < \delta_0$ and $\text{area}(M'_0) \geq 2s_0\varepsilon$. In fact one can draw a curve E'_1 like rectangle without bottom under the level $H' = \varepsilon$ above $s' - \text{axis}$ between γ'_1 and γ'_2 on (s', H') -plane. Let $E_1 = G(E'_1)$ and $\{E'_i\}_{i=1}^n = G^{-1}(E_1)$, one can draw similar

graph curve F'_2 over $s' - axis$ below E'_2 between γ'_2 and γ'_3 on (s', H') -plane. We do this similarly n times. Then, we connect E'_1, F'_2, \dots , etc. below γ'_i above $s' - axis$ to get a graph curve Γ . Finally, we close Γ with $s' - axis$ to get E'_0 . Let $L = G(\mathcal{L} \times E'_0)$. So, L is again a Lagrangian submanifold. The Lemma 3.2 still holds in this case.

3.3 Gromov's figure eight construction

First we note that the construction of section 3.1 holds for any symplectic manifold. Now let (M, ω) be an exact symplectic manifold with $\omega = d\alpha$. Let $\Sigma = H^{-1}(0)$ be a regular and close smooth hypersurface in M and H is $T - finite$. H is a time-independent Hamilton function. Set $(V', \omega') = (M \times M, \omega \oplus \omega)$. If there does not exist any close orbit for X_H in (Σ, X_H) , one can construct the Lagrangian submanifold L as in section 3.1, let $W' = L$. Let $h_t = h(t, \cdot) : M \rightarrow M$, $0 \leq t \leq 1$ be a Hamiltonian isotopy of M induced by hamilton fuction H_t such that $h_1(\Sigma) \cap \Sigma = \emptyset$, $|H_t| \leq C_0$. Let $\bar{h}_t = (id, h_t)$. Then $F'_t = \bar{h}_t : W' \rightarrow V'$ be an isotopy of Lagrangian embeddings. As in [8], we can use symplectic figure eight trick invented by Gromov to construct a Lagrangian submanifold W in $V = V' \times R^2$ through the Lagrange isotopy F' in V' , i.e., we have

Proposition 3.1 *Let V' , W' and F' as above. Then there exists a weakly exact Lagrangian embedding $F : W' \times S^1 \rightarrow V' \times R^2$ with $W = F(W' \times S^1)$ is contained in $M \times M \times B_R(0)$, here $4\pi R^2 = 8C_0$ and*

$$l(V', W, \omega) = \text{area}(M'_0) = A(T). \quad (3.10)$$

Proof. Similar to [8, 2.3B'_3].

Example. Let M be an open manifold and $(T^*M, p_i dq_i)$ be the cotangent bundle of open manifold with the Liouville form $p_i dq_i$. Since M is open, there exists a function $g : M \rightarrow \mathbb{R}$ without critical point. The translation by $tTdg$ along the fibre gives a hamilton isotopy of $T^*M : h_t^T(q, p) = (q, p + tTdg(q))$, so for any given compact set $K \subset T^*M$, there exists $T = T_K$ such that $h_1^T(K) \cap K = \emptyset$.

4 Proof on Theorems

Take $T_0 > 0$ such that $A(T_0) \geq 100\pi r_0^2$. Assume that on $H^{-1}(0)$ there does not exist periodic orbit with period $T \leq 100T_0$ and H as in section 3, then by the results in section 3, we have a close Lagrangian submanifold $W = L$ or $W = F(W' \times S^1)$ contained in $V = M \times C \times M \times B_{r_0}(0)$ or $V = M \times M \times B_{r_0}(0)$. By Lagrangian non-squeezing theorem, i.e., Theorem 2.1, we have

$$A(T_0) \leq \text{area}(M_0) = l(V, W, \omega) \leq 2\pi r_0^2. \quad (4.1)$$

This is a contradiction. This contradiction shows that there is a periodic orbit with period $T \leq 100T_0$. This completes the proofs of theorems.

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